

University of Nottingham
Department of Mechanical, Materials and Manufacturing
Engineering

Computer Modelling Techniques

FE-04-02

PRACTICAL FE EXAMPLES

Perforated Plate Example

$$L = 100 \text{ mm}$$

$$H = 50 \text{ mm}$$

$$t = 5 \text{ mm}$$

$$\sigma_o = 100 \text{ MPa}$$

The **objective of the analysis** is to determine the stress concentration around the hole.

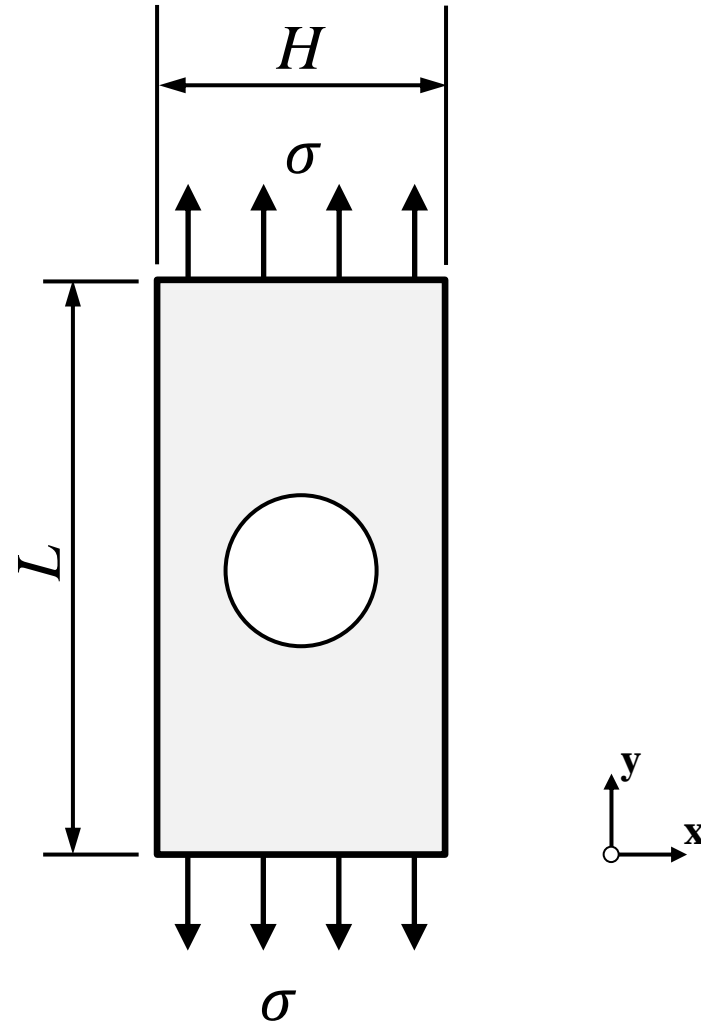


Figure 12: Perforated Plate Subject to Uniaxial Stress

Geometry

Since the plate thickness (in the z-direction) is small, **2D plane stress** conditions are applicable.

The plate (both geometry and loads) is **symmetrical** about the horizontal and vertical axes.

Therefore, **only a symmetrical quarter-model** needs to be modelled.

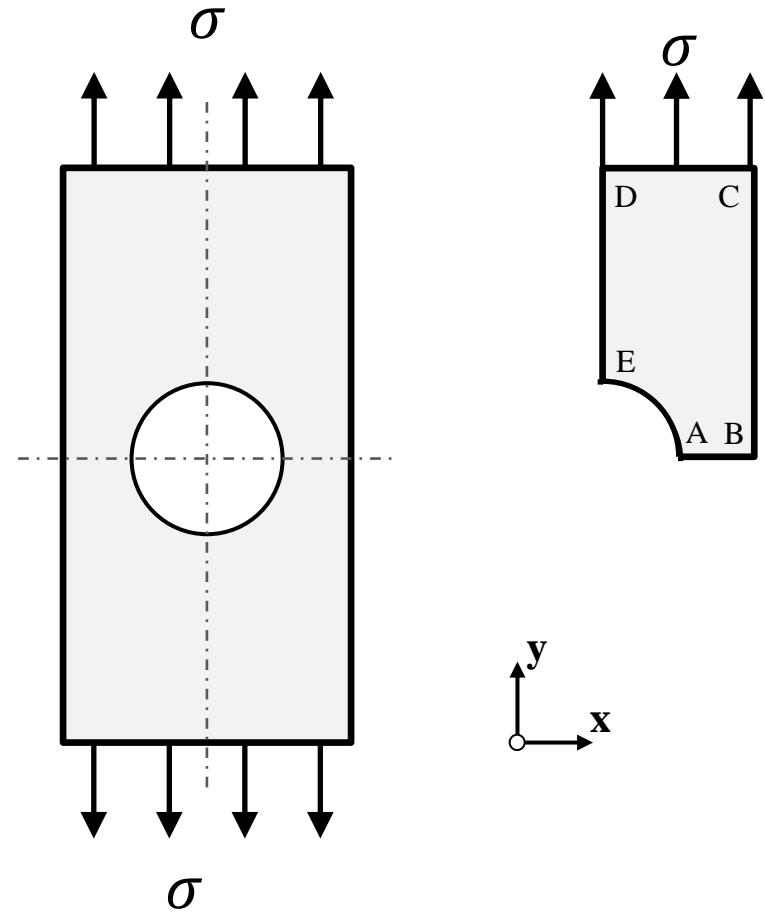


Figure 13: Symmetrical quarter of the perforated plate

Material Properties

Assuming a **linear elastic analysis**, the material properties needed are

$$E = 70 \text{ GPa}$$

$$\nu = 0.3$$

If the load is high enough to cause local **plasticity** around the hole, the elasto-plastic stress-strain curve, or at least the yield stress (σ_{yield}) must also be specified.

Boundary Conditions

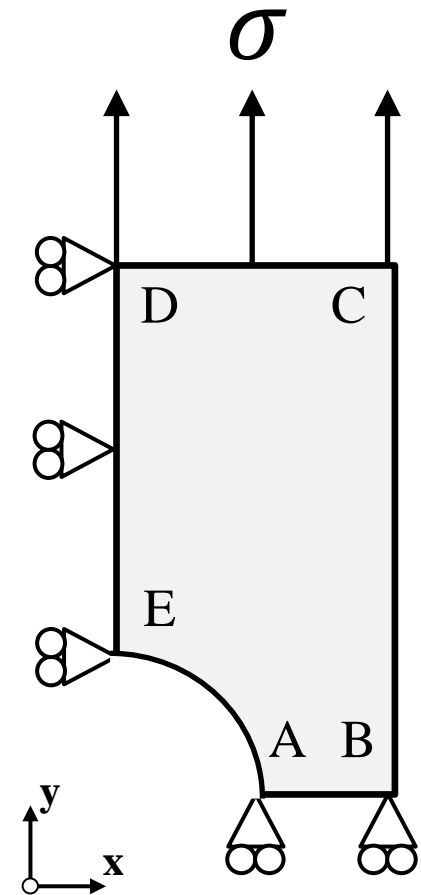
Displacement Boundary Conditions

On the axes of horizontal and vertical symmetry;
the nodes can only slide along the symmetry lines.

- (a) Zero y-displacements (roller conditions) specified on line AB.
- (b) Zero x-displacements (roller conditions) specified on line DE.

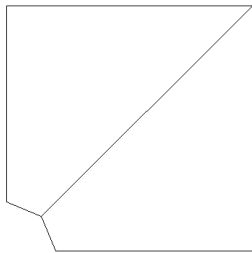
Applied Loads

A uniform tensile stress (distributed load), σ_0 , is specified at the top surface (line CD).

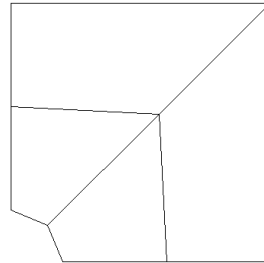


FE Model

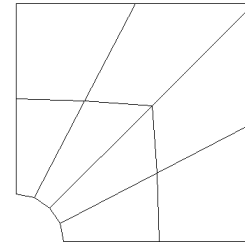
- **2D plane stress** linear (4-node) or quadratic (8-node) elements can be used here.
- Either **quadrilaterals or triangles**, or a combination of the two, can be used.
- **Quadratic elements** are suitable for this problem, since they can represent the circular hole geometry better than linear elements.
- Since stress concentration is expected around the hole, **mesh biasing** should be specified around the hole.
- If the yield stress is known, a **plasticity check** can be performed by checking the maximum value of the effective (von Mises) stress.



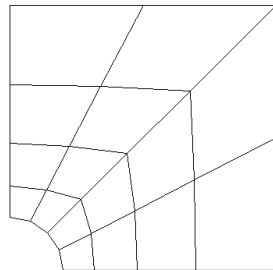
Mesh A: 2 Elements



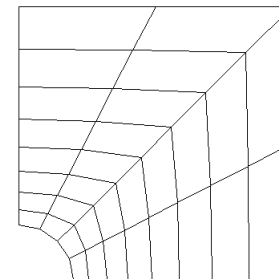
Mesh B: 4 Elements



Mesh C: 8 Elements



Mesh D: 16 Elements



Mesh E: 32 Elements

Figure 14: FE meshes used for the perforated plate example

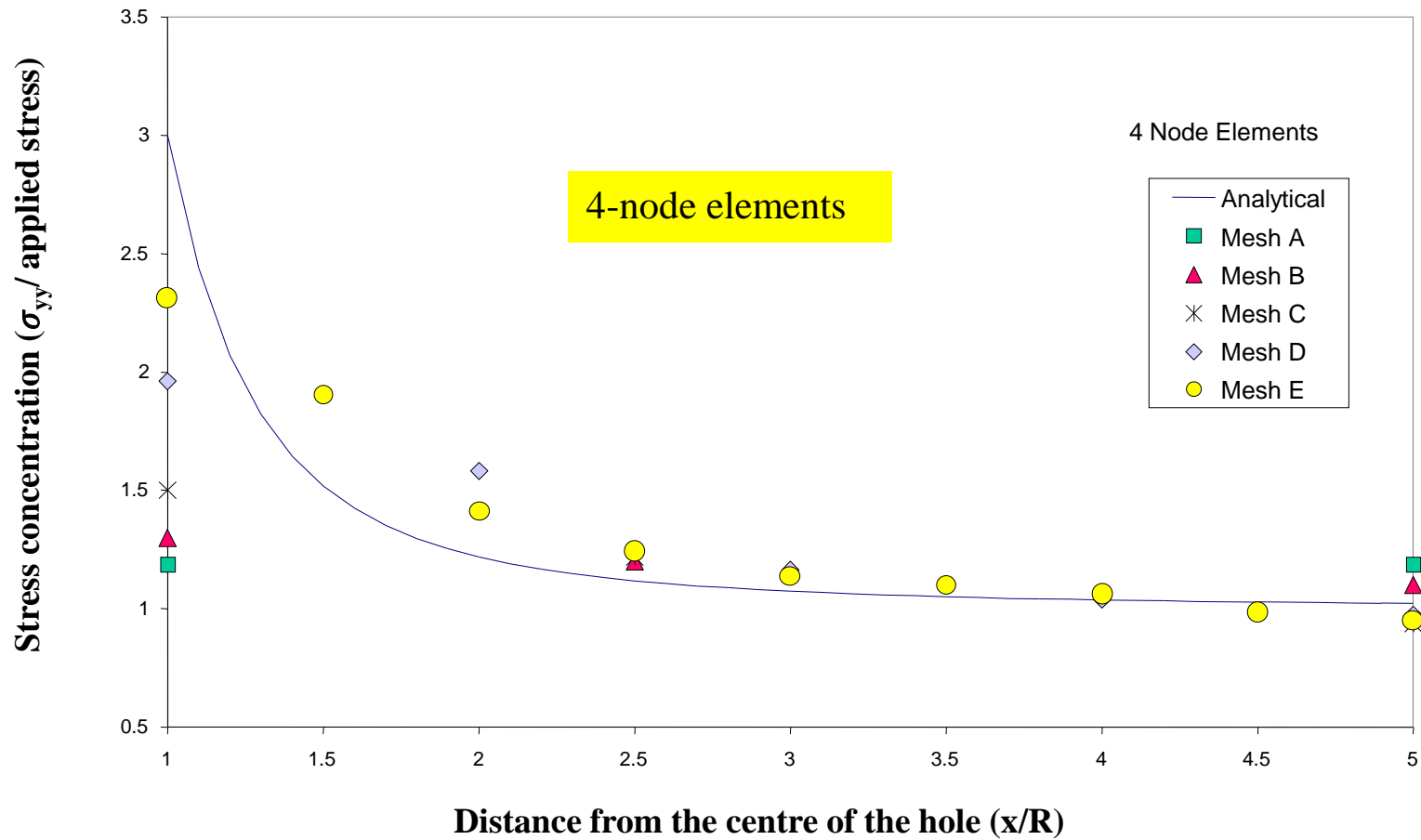


Figure 15: Comparison of FE and analytical solutions for the perforated plate example (4-node 'linear' elements)

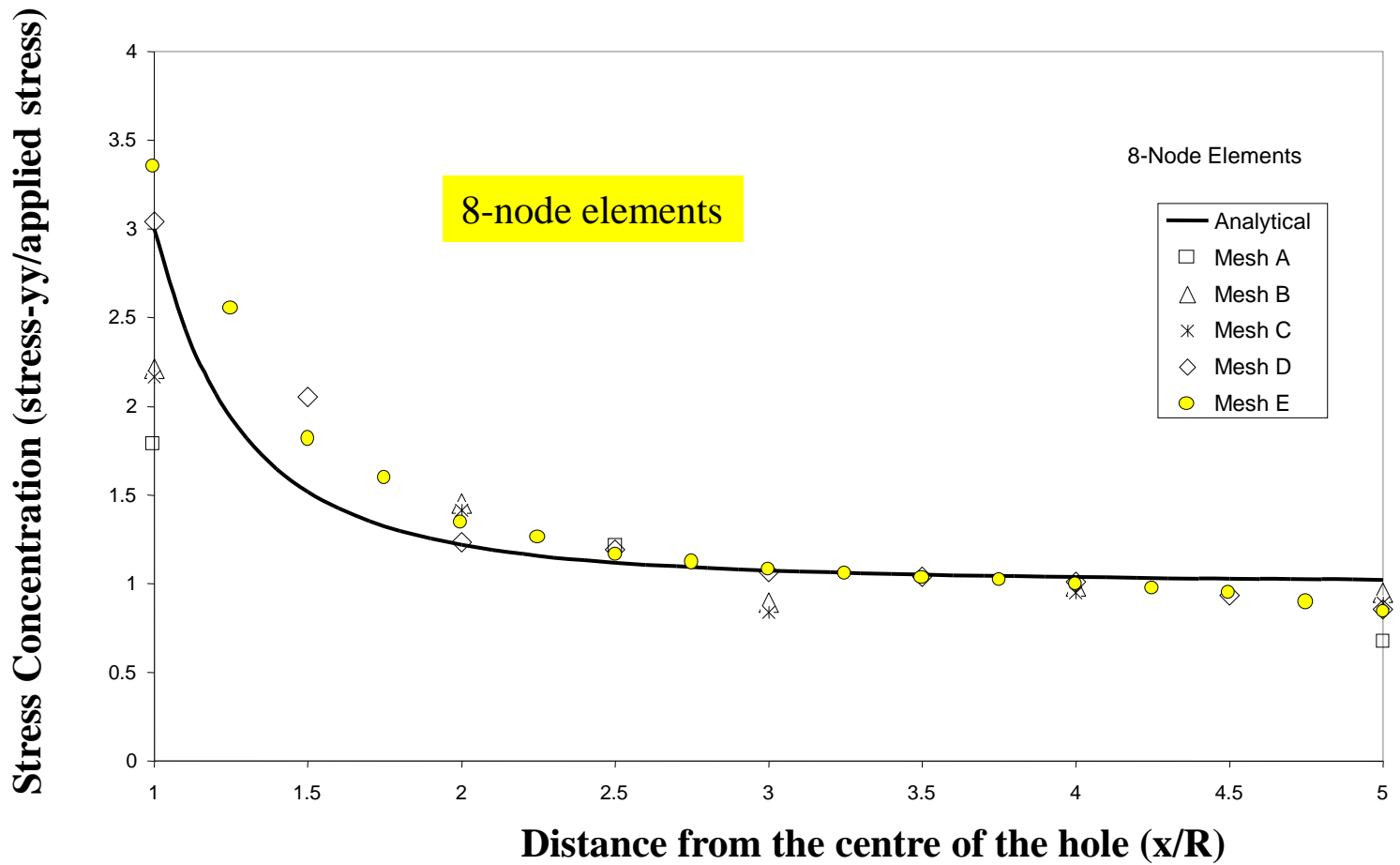


Figure 16: Comparison of FE and analytical solutions for the perforated plate example (8-node 'quadratic' elements)

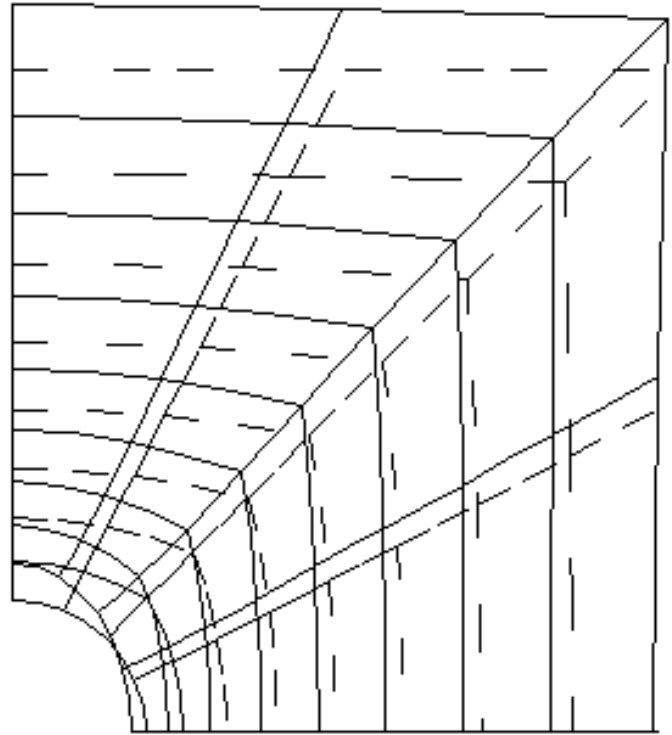


Figure 17: Exaggerated deformed shape (solid lines) for the perforated plate example

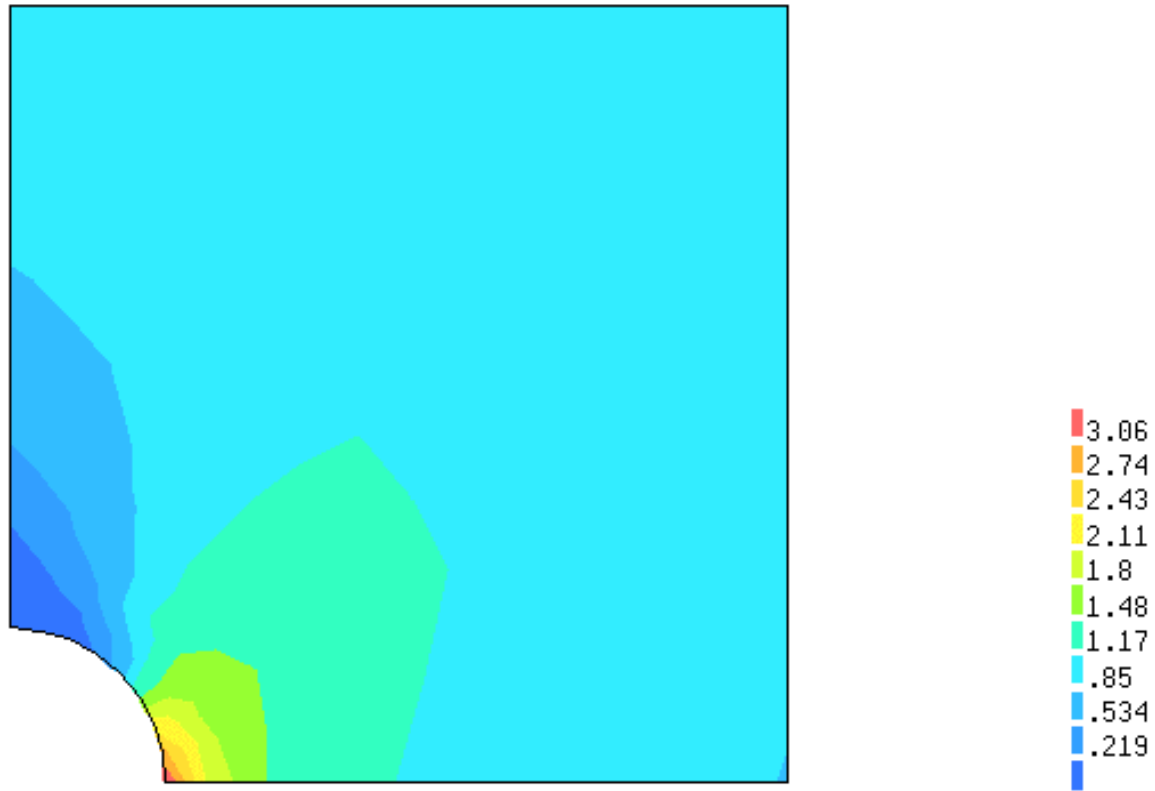


Figure 18: Stress contour plot (σ_{yy}) for the perforated plate example

Cantilever Beam Example

Problem Definition

The beam has a square cross-sectional area of side length t .

$$L = 2 \text{ m}$$

$$t = 0.1 \text{ m}$$

$$F = 1 \text{ kN}$$

The objective of the analysis is to obtain the overall **deflection** of the beam.

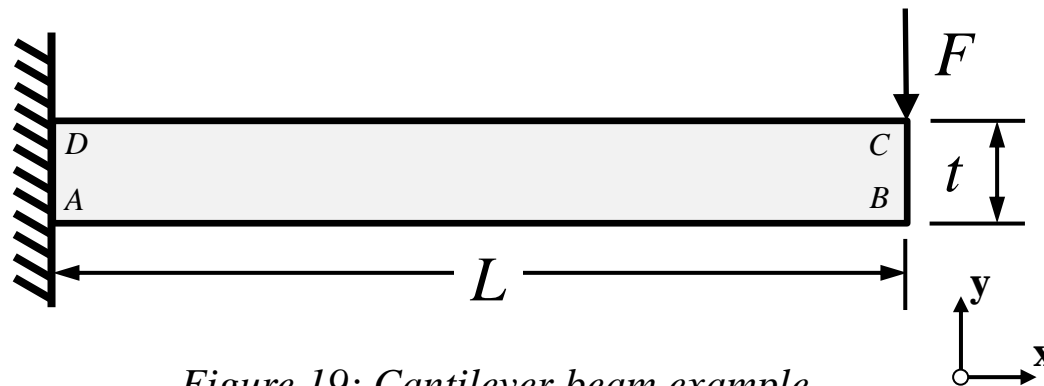


Figure 19: Cantilever beam example

Geometry

- Since there is **no symmetry** in this problem, the whole geometry has to be modelled.
- The geometry can be modelled with **beam elements** since the geometry and loads satisfy beam bending conditions, i.e. the geometry is long, slender and subjected to only transverse loads.
- It is also possible to model this problem with **2D plane stress elements** since the thickness in the z-direction is sufficiently small.

Material Properties

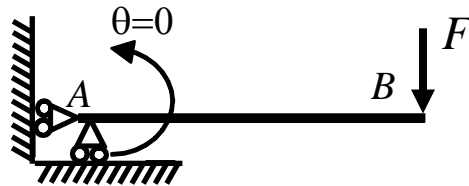
Assuming a linear elastic analysis

$$E = 200 \text{ GPa}$$

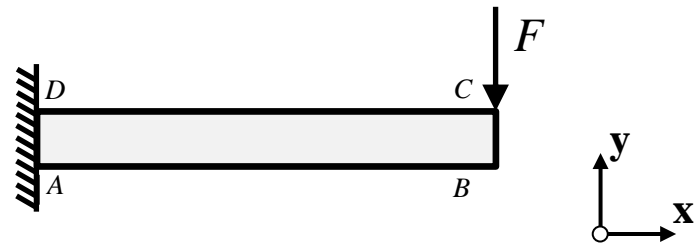
$$\nu = 0.3$$

Boundary Conditions

- The cantilever is built-in at the left hand side.
- If **beam elements** are used, then **both the displacement and the slope** (gradient of the displacement) at the built-in node must be prescribed as zero.
- If **2D plane stress elements** are used, then all nodes on line AD must have zero displacements in the x and y directions, which automatically enforces the built-in condition.



(a) Beam FE model



(b) 2D continuum FE model

Applied Loads

A point load of magnitude F is applied to point C.

If a 2D plane stress model is used, this point force can either be applied at point C, or distributed along the line BC.

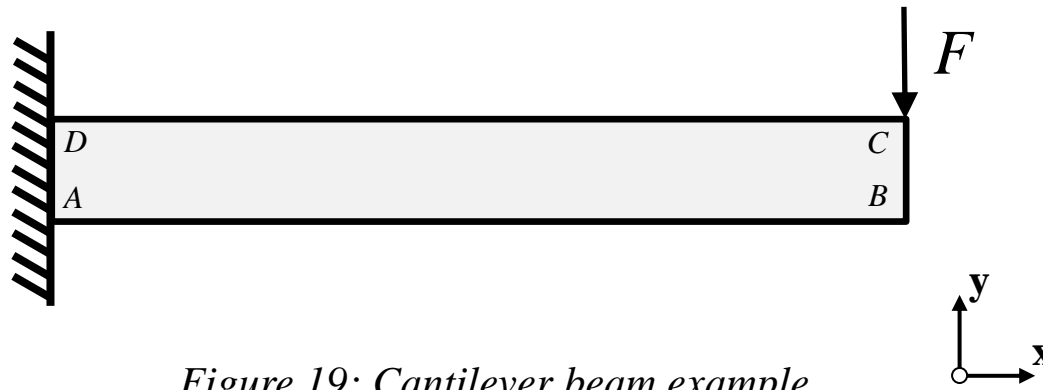


Figure 19: Cantilever beam example

FE Model

Two types of elements can be used to model this problem;
Beam elements or **2D plane stress elements**.

Of course, it is always possible to model this problem using **3D elements**, but that would be unnecessary.



3-node beam element mesh (4 elements)

2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31

8-node 2D plane stress elements (32 elements)

Figure 20: FE meshes used for the cantilever beam problem

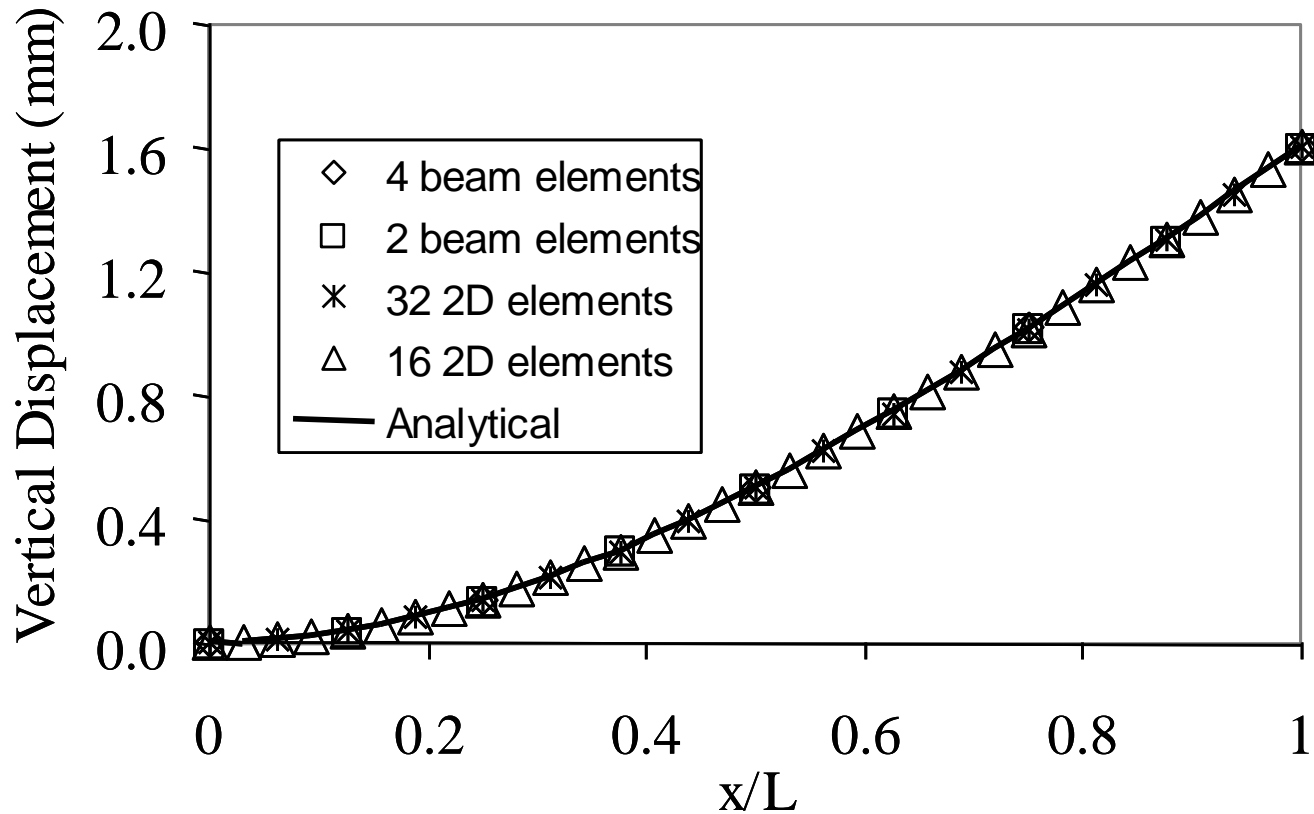
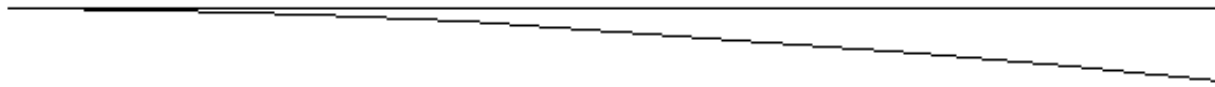


Figure 21: Comparison of FE and analytical solutions for the cantilever beam problem



Beam element mesh



2D plane stress element mesh

Figure 22: Deformed shapes (solid lines) for the cantilever beam problem

Key Points for FE

- FE analysis, **specify the geometry, material properties, analysis type, displacement boundary conditions and applied loads.**
- **Question** the inputs and assumptions and their sensitivity to your model
- **Simplify** – Reduce the size & complexity of your problem
 - Reduce complexity of your model (geometry, physics, loading conditions)
 - Reduce dimensions of your problem (3D → 2D)
 - Apply symmetry
- **Check** carefully FE solutions – they are and not taken for granted to be accurate.
- **Perform mesh convergence** studies on your analyses to have confidence in FE accuracy



Validate your models either analytically or experimentally

References for Commercial FE Software

On your future engineering journey you will hear about many different FE analysis software.



MSC Nastran



Marc★

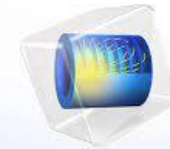


AUTODESK★
FUSION 360™



LS-DYNA®

COMSOL
MULTIPHYSICS®



Altair OptiStruct™★

Altair Inspire™★

Ansys



PrePoMax

CalculiX

However, what we learnt applies to all the software!

★ Student edition available